ANALYSIS OF CHAOTIC MOVEMENTS OF A BALLOON IN MARTIAN ATMOSPHERE

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ABSTRACT.

I analyze the movement of a balloon in Mars's atmosphere, while encountering an inversion layer. An inversion layer model with a non-monotonic density versus height variation is considered. Using a program that simulates the movement, I have determined that chaotic movements may occur. Chaotic movements may significantly modify the balloon's trajectory, moreover may induce high tensions in the balloon's structure.

INTRODUCTION

We analyze the movement of a balloon in the Martian atmosphere, while encountering inversion layers. Inversion layers represent non-monotonic variations of the density with the height and therefore do not follow the standard atmospheric model. Inversions are frequent in Mars's atmosphere. Frictions are not taken into consideration. Chaotic movements may appear when a balloon passes through an inversion layer. The consequence of chaotic movements is that they may significantly alter the balloon's trajectory, moreover induce high strains in the balloon's structure. The goal of my paper is to determine when chaotic movements occur.

ATMOSPHERIC MODEL DISCUSSION

For the analysis the typical Mars atmospheric model is used. A theoretical model has been proposed for the non-monotonic variation of the density with the height of the inversion layer. The usual Mars atmospheric model from NASA Glenn Research Center can be found at http://www.lerc.nasa.gov/WWW/K-12/airplane/atmos.html. The general law of variation of the density with the height is for Mars's atmosphere [1]:

$$\rho(h) = \frac{0.699 \cdot e^{-0.00009 \cdot h}}{0.1921 \cdot (-31 - 0.000998 \cdot h + 273.1)}$$

The calculus is made without considering the frictions of the balloon with the gases or dust from the atmosphere. I have used a program that shows the non-linear variation of the density of the atmosphere with the height for the Martian and terrestrial atmosphere (see Fig. 1). The density values are computed as varying with the height for a maximum height of 7000m.

DYNAMIC ANALYSIS

The equilibrium condition for the balloon is:

$$\vec{F}_{Archimede} = -\vec{G}$$

The force of Archimedes is equal in modulus with the weight of the balloon. A representation of the forces that act upon the balloon is shown in figure 2.

The forces acting upon the balloon are the force of Archimedes $\vec{F}_{Archimedes}$, the weight $\vec{G}$, and the inertia force $\vec{F}_{Inertia} = -m \cdot \vec{a}$ (the inertial force is taken into account for an accelerated movement). The gravitational acceleration constant for Mars that we use is the mean value of the gravitational acceleration at the poles and at the equator and is $\vec{g} = 3.7375 \left[m/s^2 \right]$.

The equilibrium condition is:

$$-\vec{G} = \vec{F}_{Archimede} \Leftrightarrow \vec{G} + \vec{F}_{Archimede} = 0 \Leftrightarrow V_{balloon} \cdot \rho_{balloon} \cdot \vec{g}_{Mars} = V_{balloon} \cdot \rho(h_0) \cdot \vec{g}_{Mars}$$
$$\Rightarrow \rho_{balloon} = \rho(h_0)$$

From the equilibrium condition the equilibrium height h_0 is determined. If the balloon has an initial speed, we add a term $h_{initial} = v_{initial} \cdot t$. Such a case appears when a satellite launches the balloon into Mars's atmosphere.

Assuming that the balloon is oscillating around the equilibrium position with a distance *x* (from h_0), then the following equation will be considered, $x = h - h_0$.

By determining *x*, and knowing the height at which the balloon is, the equilibrium height h_0 is determined from $h = h_0 + x$. The movement equation is:

$$-m\ddot{x}(t) - \vec{F}_{Archimede} + \vec{G}_M = 0 \Leftrightarrow -m\ddot{x}(t) - \rho(h_0 + x) \cdot V \cdot \vec{g}_M + \vec{g}_M$$
$$\Leftrightarrow \ddot{x}(t) - \rho(h_0 + x) \cdot \frac{V \cdot \vec{g}_M}{m} + \vec{g}_M = 0 \Leftrightarrow \ddot{x}(t) + f(x) = 0 \qquad (1)$$

where G_M is the weight of the balloon and we denote by

$$f(x) = -\rho(h_0 + x) \cdot \frac{V \cdot \vec{g}_M}{m} + \vec{g}_M \qquad (2).$$

From the equations (1) and (2), it follows that

$$\ddot{x}(t) = f(x) \qquad (3).$$

INVERSION LAYER MODEL

The inversion layer is modeled by a non-monotonic profile of density, that is, the density of the inversion layer is varying nonlinearly with the height. A parabolic equation has been used to model the variation with the height of the inversion layer's density:

$$\rho = \rho(h) = \rho_0(h) + ah \cdot (1 - h) \approx ah \cdot (1 - h)$$

where *a* is a constant, *h* the height; the approximation is valid inside the inversion layer.

The graphical representation of the variation of the density with the height for the Martian atmosphere is shown in Fig. 3 and represents the density of the inversion layer varying according to a parabola.

APPROXIMATION OF DERIVATIVES FOR NUMERIC CALCULUS

To be able to solve the movement equation, I have written a C program that computes the values of $x(t)$ and $\dot{x}(t)$. The program simulates the movement by doing a space versus time graph and the phase diagram of the movement. The phase diagram is a representation of the movement in space versus speed graph. If the phase diagram shows a strange attractor, then the movement is chaotic. I recall that a strange attractor is not a closed curve, but a curve that fills an area of the space.

To compute the derivatives, I had to discretise them. Due to the nonlinearity of the movement equation, chaotic movements may appear. Chaotic movements may induce high tensions in the balloon's structure and may significantly modify the balloon's trajectory.

We discretise the derivatives to solve numerically the equation (3):

$$\dot{x}(t) = \frac{x(t) - x(t - \Delta)}{\Delta} \;;$$

$$\ddot{x}(t) = \frac{\dot{x}(t) - \dot{x}(t - \Delta)}{\Delta} = \frac{\dfrac{x(t) - x(t - \Delta)}{\Delta} - \dfrac{x(t - \Delta) - x(t - 2 \cdot \Delta)}{\Delta}}{\Delta} = \cdot$$

$$= \frac{x(t) - 2x(t - \Delta) + x(t - 2 \cdot \Delta)}{\Delta^2}$$

For the discretization of the derivatives we take into account that $\Delta \to 0$. In the program, Δ is reduced to a scaling factor. By using the discretization method, we change in the equation (3) $\ddot{x}(t)$ with its discretized form and we divide the equation by Δ^2:

$$x(t) = f(x) + 2x(t - \Delta) - x(t - 2 \cdot \Delta) \Leftrightarrow x(t) - f(x) = 2x(t - \Delta) - x(t - 2\Delta)\,.$$

We denote by "expression"$= x(t) - f(x) - 2x(t - \Delta) + x(t - 2\Delta)$.

"$\exp ression$"$= 0$ is the movement equation.

The function *f(x)* is the function that determines $\rho(h)$ and is computed in the program.

The time-dependent function $x(t)$ is computed in a "*for*" loop. $x(t)$ will be computed from the formula for each moment of time.

NUMERICAL SOLVING OF THE MOVEMENT EQUATION AND RESULTS

The density of the inversion layer is modeled by the equation:

$$\rho_2 = \rho_{20} \cdot f(h) = \rho_1(h_0) \cdot \left(a_0 + a_1 \cdot x + a_2 \cdot x^2\right)$$

where the distance from the equilibrium position, x, satisfies the condition $x > 0$, and a_1, a_2, a_3 are constants. As an example, I used $a_0 = 2$; $a_1 = 0.3$; $a_2 = -0.01$. We denote by:

$$\alpha = \frac{S \cdot g \cdot \left[\rho_1(h_0) \cdot \left(2 - 0.01x^2 + 0.3x\right) - \rho_1(h_0)\right]}{m} = \alpha_0 \cdot \left(2 - 0.01x^2 + 0.3x\right)$$

The movement equation is:

$$\ddot{x}(t) = -\alpha(h) \cdot x(t) = -\alpha_0 \cdot \left(2 - 0.01x^2 + 0.3x\right) \cdot x(t) \qquad (4)$$

We use the discretized forms of the derivatives in the movement equation (4) and we obtain - considering we have only the values at the anterior moments:

$$x(t) = \frac{2 \cdot x(t-1) - x(t-2)}{1 + \alpha \cdot \left(1 - 0.01 \cdot x(t-1)^2 + 0.3 \cdot x(t-1)\right) \cdot \Delta^2} \qquad (5)$$

The following code lines have been used for the computation of $x(t)$:

```
x[0]=1.0;  // initial values
x[1]=1.2;
for (t=2;t<5000;t=t+1)
  {
    x[t]=0.;   //initialization at 0
  }
printf("\n alpha=%5.3f \t", alpha);  //showing alpha on the screen
for (t=2;t<N;t=t+1)    //N is defined as the number of steps
 { //parameter = 0.01;
    x[t]  =(2.*x[t-1]-x[t-2])/(1.+ alpha*(1-parameter*x[t-1]*x[t-1]+ \
     0.3*x[t-1])*delta*delta);   //computation formula of x(t)
     printf("x[%d]=%3.3f \t",t, x[t]); //showing obtained x(t)values
    getch( );
 }
```

The resulted movement is obtained by simulation using the program for the following values: $\Delta = 0.01$ and parameter = 0.01. A sample result is shown in Fig. 4.

For the phase diagram we need the values of the speed and of the space. The phase diagram is representing the movement point-with-point for different moments of time. For the phase diagram, I have used the following code lines:

```
line(30, 400, 400, 400); // vertical axis, Oy
line(30, 400, 30, 100); //horizontal axis Ox
outtextxy(180, 420,"x(t)");
outtextxy(20, 150, "v(t)");
outtextxy(100, 450,"Phase diagram");

for(t=50;t<N-1;t=t+1) //here I execute the phase diagram
{//here I execute the graphic;
   line(200+(int)(0.01*x[t-1]),300-(int)(0.004*v[t-1]),
         200+(int)(0.01*x[t]),_300-(int)(0.004*v[t]));
}
 getch();
```

During the analysis, I have observed that for different values of the parameters of the equation different movements will be obtained. Chaotic movements and unstable movements (to which their amplitude is increasing permanently) have been obtained. The values of the parameters of the equation are important, because small variations of the parameters may stabilize the movement or change it to a periodical movement.

In the Figure 5, some results of chaotic movements are shown. The movements are represented in phase diagrams and space versus time diagrams.

In Figure 6, a phase diagram of the movement is illustrated. The phase diagram represents a chaotic movement, because it is a strange attractor, that is a curve that "fills" some region of the space, not a closed curve.

CONCLUSIONS

I analyzed the movement of a balloon in Mars's atmosphere, while encountering an inversion layer. I have used an atmospheric model that includes an inversion layer, the density of the inversion layer non-monotonically varying with the height. Using a program, the movement has been simulated and I determined that chaotic movements might appear. Such movements may significantly modify the balloon's trajectory and in some cases there are gains of high speeds in small amounts of time, therefore inducing high tensions in the balloon's structure. The analysis applies not only for a balloon, but also for any floating object in Mars's atmosphere.

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REFERENCES

[1] Mars (usual) atmospheric Model, Tom Benson, NASA Glenn Research Center, http://www.lerc.nasa.gov/WWW/K-12/airplane/atmos.html

[2] "Mars Atmospheric Profiles, Inversion Layers and Derivation of the Balloon Equilibrium Height", Horia-Mihail Teodorescu, ARCHIMEDES Mars Balloon Mission Documentation, Mars Society Deutschland, 2004

[3] "Analysis of nonlinear dynamics of a balloon in Martian atmosphere encountering an inversion layer", Horia-Mihail Teodorescu, Proceedings 3rd European Mars Conference, 2003

FIGURES

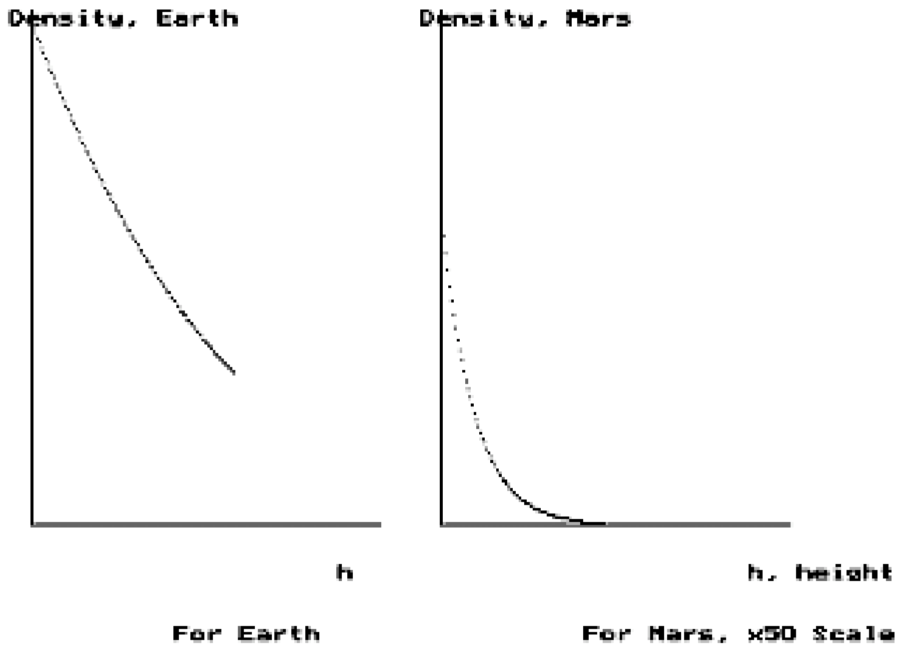

Fig. 1. Graph showing the non-linear variation of the density with the height for the Martian and Terrestrial atmosphere, for 100 iterations (simulated profile)

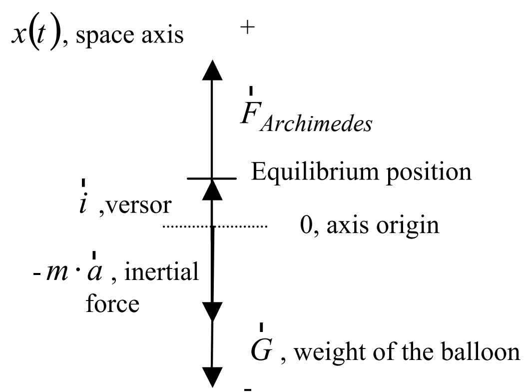

Fig. 2. Representation of the forces that act upon the balloon.

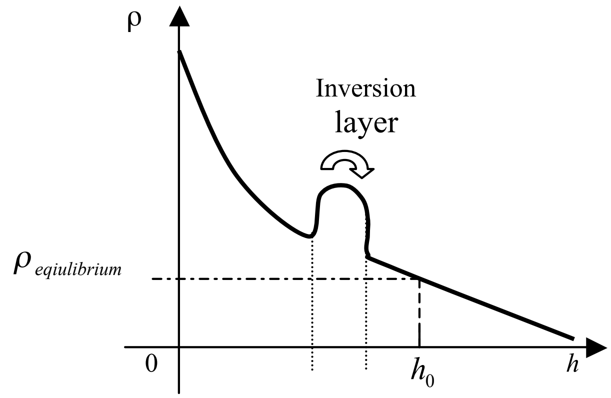

Fig. 3. Representation of the nonlinear variation of the density with the height for the atmospheric model that takes into account the inversion layer.

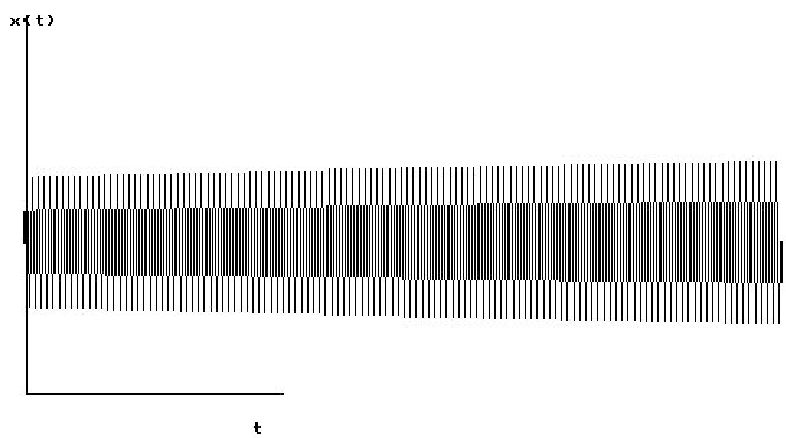

Fig. 4. The movement is represented for the variation of $x(t)$ with the time *t*. The resulted graph shows an oscillatory movement that auto amplifies.

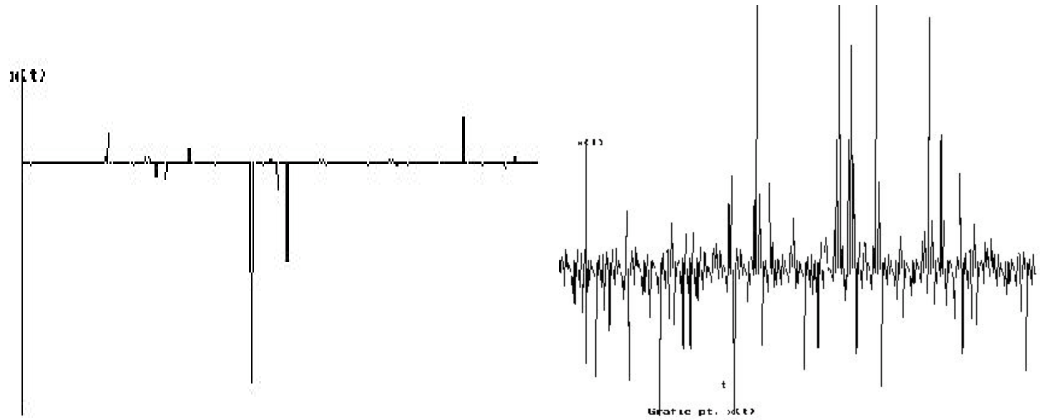

Fig. 5. Representation of a chaotic movement in a space versus time diagram. The first panel is for the first 500 iterations, while the last panel is for the last 500 iterations. Both panels represent the same movement, at different moments of time.

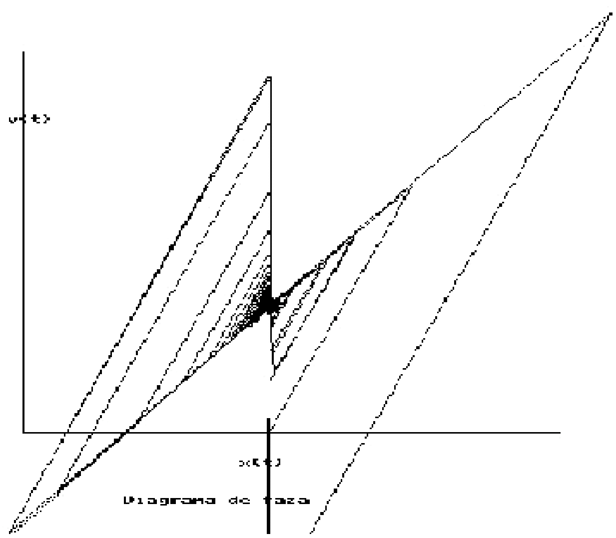

Fig. 6. Phase diagram representing a chaotic movement.