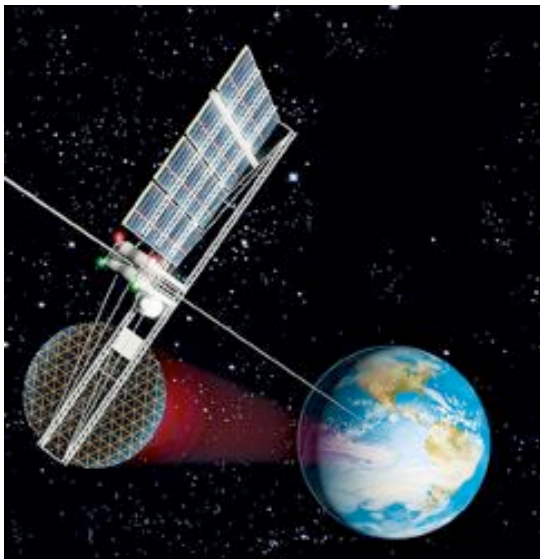


# **The Weakest Link in the Space Elevator Vision\* - Carbon Nanotubes**

**Dr. Frank W. Crossman**

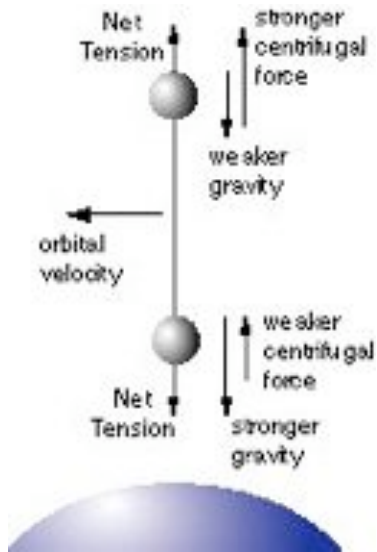
**Palo Alto, CA**



**\*Humans to orbit  
at a cost of  
~\$100 per lb**



## Momentum Exchange Space Tethers reduce the energy needed to reach orbit velocities

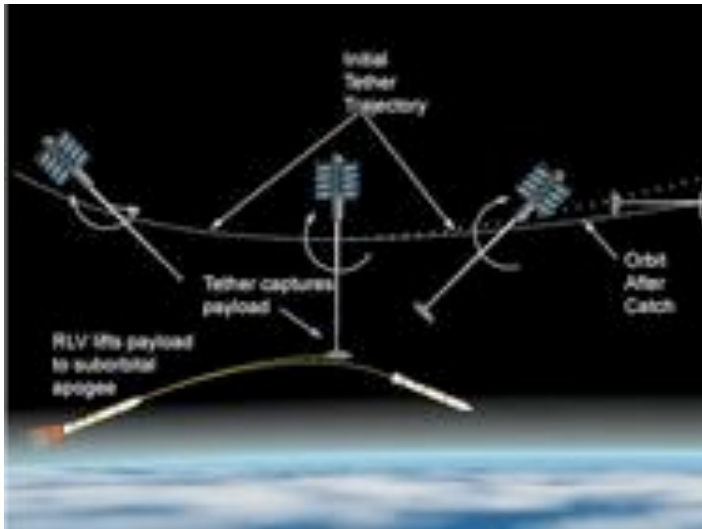


A **stationary tether** is one that connects two masses together and remains at constant length. If the tether and its masses are orbiting a massive body, then typically the system will be gravity gradient stabilized, with the tether pointed along the radius vector to the massive body. A tether with CM at GEO and reaching to the surface of the earth is called a

- **beanstalk** or **space elevator**.

A stationary tether that reaches down to the upper edge of the atmosphere is called a

- **skyhook** or “**hypersonic skyhook**” since the relative velocity of the lower tip with respect to earth is a typically hypersonic  $\sim$  Mach 15.



- A **bolo** is a long rotating cable anywhere in space that is used as a "momentum-energy bank".

- A **rotovator** is a long bolo in low orbit around a planet (or moon) that is used to reach down from space to lift payloads from rocket planes near a planet's surface. The rotating tip can lower the relative velocity from that of the stationary skyhook concept to  $\sim$  Mach 10.

# The Hypersonic Stationary Skyhook\* Taper equation

\* R. Zubrin (1996)

A constant stress tether will be thickest at its center of mass (cm) and taper to a minimum cross section at its tip. The ratio of the thicknesses is the taper ratio.

$$\text{Taper ratio} = \exp \left[ \left( \frac{L}{U^2} \right) \left\{ \left( g_1 R_1 / (R_1 + L) \right) - \mu (R_1 + L/2) / (R_1 + L)^3 \right\} \right]$$

Where  $\mu$  is the gravitational parameter of the planetary body =  $G \cdot m_p$

$g_1$  is the acceleration due to gravity at the base of the tether

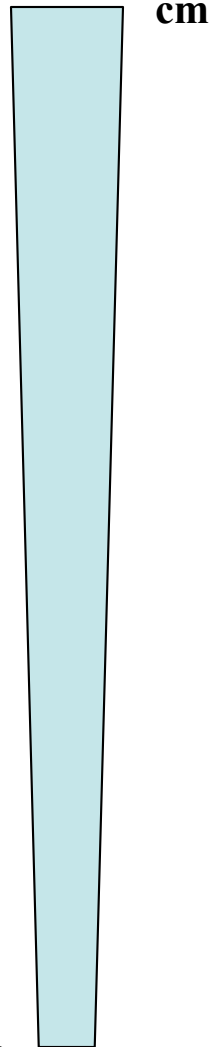
$R_1$  is the radius from planet center to the **base of the tether**  $R_p + 100$  km

**L** is the length of the tether

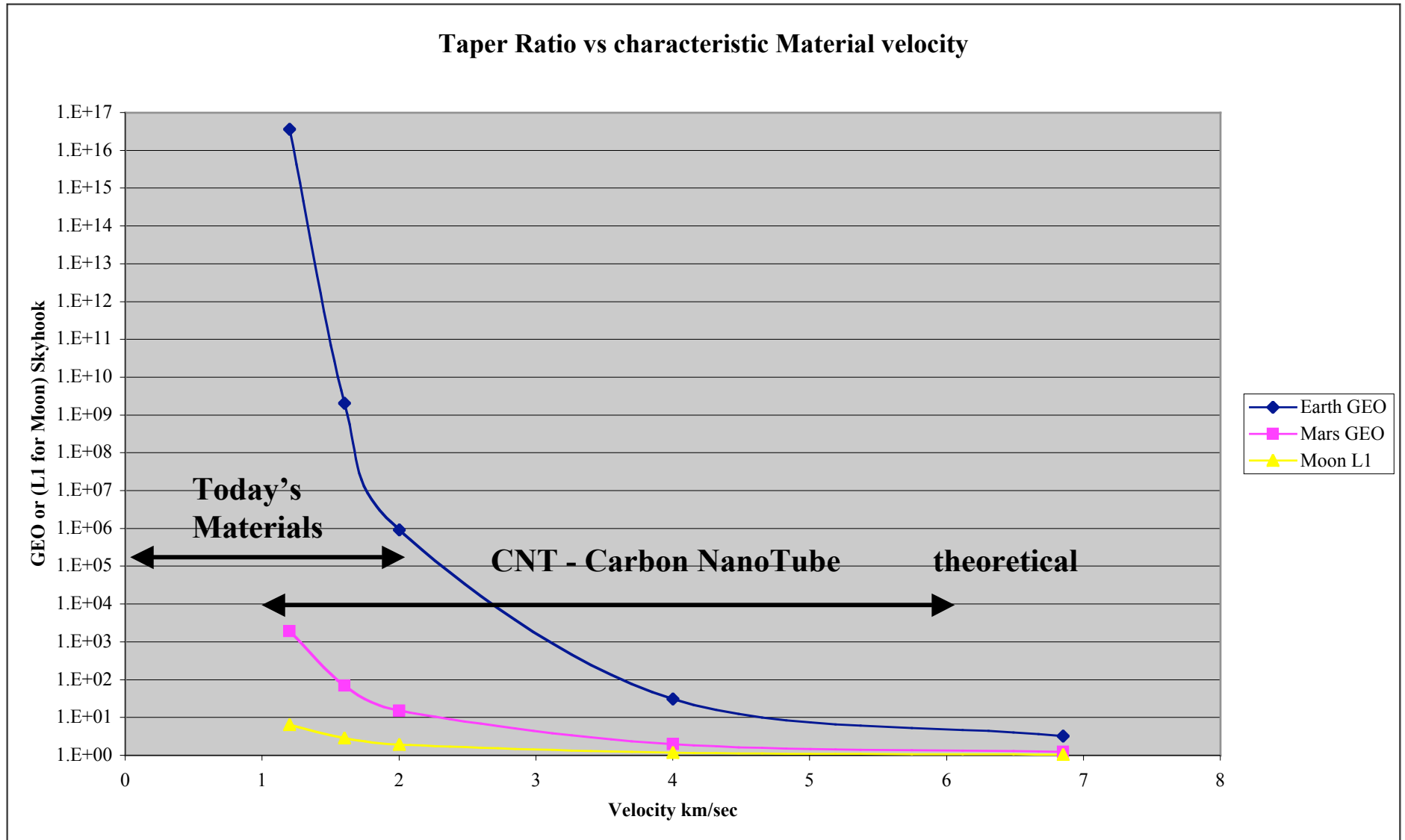
**U** is the characteristic velocity of the material =  $\sqrt{(\sigma_m / \rho_m)}$

Parameters	Earth	Mars	Moon
$\mu$ km <sup>3</sup> /sec <sup>2</sup>	3.98601E+05	4.28280E+04	4.90279E+03
$g_1$ km/sec <sup>2</sup>	9.81000E-03	3.71000E-03	1.62000E-03
$R_p$ km	6.37800E+03	3.39700E+03	1.73800E+03
$V_t$ km/sec	4.63000E-01	2.41000E-01	4.62736E-03
RGEO km	4.22411E+04	2.04241E+04	8.84054E+04
RL1 km			6.15000E+04

**Tip @ 100km above earth surface**



# GEO centered Space Elevator requires very high characteristic material velocity



# Introduction - Why this paper?

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**2003 NorCal MarsSoc meeting:** It started with a discussion with Ben Shelef (CEO, Spaceward Foundation) about Space Elevator and Carbon Nanotube properties

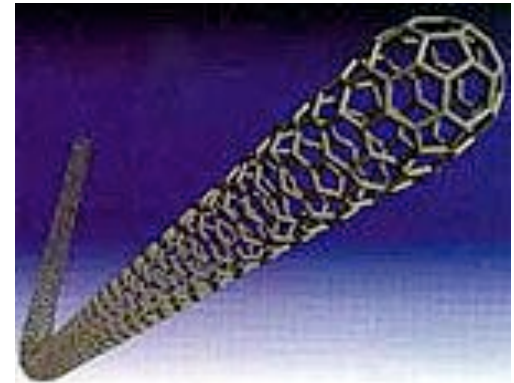
**Ben:** We have a single strength data point of 63 GPa (6.8 km/sec) for MWCNT that is sufficient to build a Space Elevator... Fibers/cable development will take 5-10 years... So, stop all this work on Mars exploration & settlement by rockets until we build the space elevator.

**I countered with:** You know materials development takes 10 years to product and 25 years to mature. Example: carbon fibers.

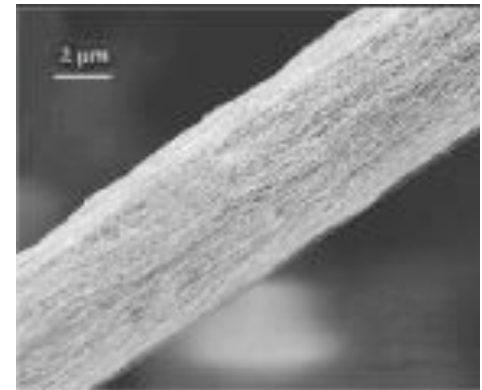
**Ben:** Nanotubes of carbon are really different from those other materials; you'll see!

**I thought but didn't say:** No, Ben, you will see. I'm the Ph. D. in Materials Science and I should know!

**But by 2006:** 4 Sessions on the Space Elevator at ISDC, CNT cables are produced, & NASA supports robotic climber tests.



**SWCNT  
~10 nm diameter**



**DWCNT cable  
2 $\mu$ m diameter**

## **... and then some comments by Burt Rutan**

---

**At the May 2006 ISDC Burt says:** If I do get to the Moon in my lifetime... it will because of breakthroughs that make the kinds of differences that we saw with suborbital manned flight capability... but **I don't know what that breakthrough technology will be that would greatly lower the cost ... of human flights to the Moon or even Earth orbit.**

I was really taken aback by this thought that human space flight cost must be reduced significantly to allow the average Earthling to spring loose from this planet.

**If Burt doesn't know the answer or even the path, who does?**

**Revisiting the Space Elevator I found its design is still based on carbon nanotube operating at a working stress close to its theoretical strength.**

**As a materials scientist I had to speak up... and that is why this paper was prepared.**

# What is the theoretical strength of CNT?

Knowing the atomic bond energy vs the distance of separation of atoms one calculates the work to separate the atoms and when the bond is broken one equates that total work to the surface energy  $\gamma$  of the two new surfaces created. **The maximum value the force (stress) needed to separate the atoms is defined as the theoretical strength  $s_{th}$**

For Force/area = stress ( $s$ );  $\Delta L/L$  = strain ( $\epsilon$ )

$E$  is the stiffness or modulus = stress/strain where

$$\sigma_{th} = \sqrt{[(E \cdot \gamma)/(n \cdot b_0)]} \quad n=1 \dots 5$$

$b_0$  is the equilibrium bond length

Surface energy  $\gamma$  is  $\sim E b_0/40$

$$\sigma_{th} = .05 \dots 0.20 E$$

The stiffnesses,  $E$  ( $= \sigma/\epsilon$ ), of the carbon  $sp^2$  bond in graphite and  $sp^3$  bond in diamond are the highest of any known material at  $\sim 1$  TPa ( $10^{12}$  Pascals) so the theoretical strength of CNT (with graphene  $sp^2$  bonds) is

$$\sigma_{th} = .05-.20 E = 50-200 \text{ GPa} \quad (V_c = 6-12 \text{ km/sec})$$

**A working stress of  $\sim 60$  GPa is needed to build the Space Elevator**



## In a perfect world, CNTs would build a Space Elevator, but it isn't a perfect world

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### A short tutorial in crystalline defects

From thermodynamics principles we find that all crystalline structures contain defects that create local stress concentrations that in turn reduce the measured strength.

$$\sigma_{\text{local}} = K \sigma_{\text{average}}, K \gg 1$$

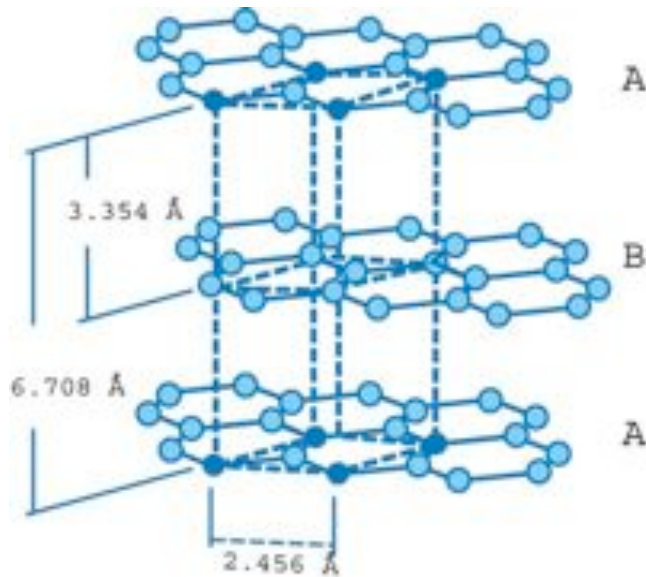
- Point defects - vacancies or impurity substitutional atoms or interstitials
- Line defects - edge & screw dislocations, disclinations
- Planar defects - small semicircular surface cracks, cracks at crystal boundaries,
- Volumetric defects - Stone-Wales bond rotation defects or  $sp^2$  -  $sp^3$  bond transition regions

Some materials are more perfect than others, ... but not perfect.

- Whiskers (sapphire, iron, graphite, silicon) have a **central screw dislocation** along the axis of the whisker and strengths ranging from 5-20 GPa
- Carbon nanotubes (CNTs) have been shown by molecular mechanics modeling to have **vacancy and Stone-Wales defects**.

# Graphite, Graphene, and Carbon Nanotubes

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**Graphene is a single atomic layer of carbon atoms very strongly covalently bonded (sp<sup>2</sup> bonds) in a planar hexagonal array.**

**Graphite is a crystal composed of graphene layers with a weak bond (Vander Waal's temporary dipoles) between the graphene layers.**

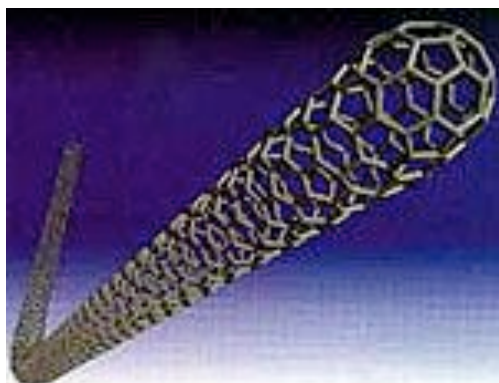
**Pentagonal and heptagonal defects are energetically easy to create in the planar graphene sheet and cause out of plane distortions.**



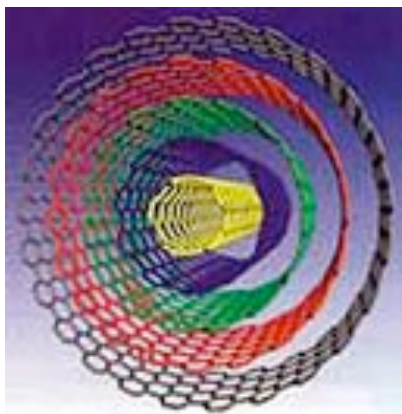
**Carbon nanotubes are graphene cylinders with a graphene cap (that includes a pentagon) at each end.**

## Nanostructural arrangements of CNTs

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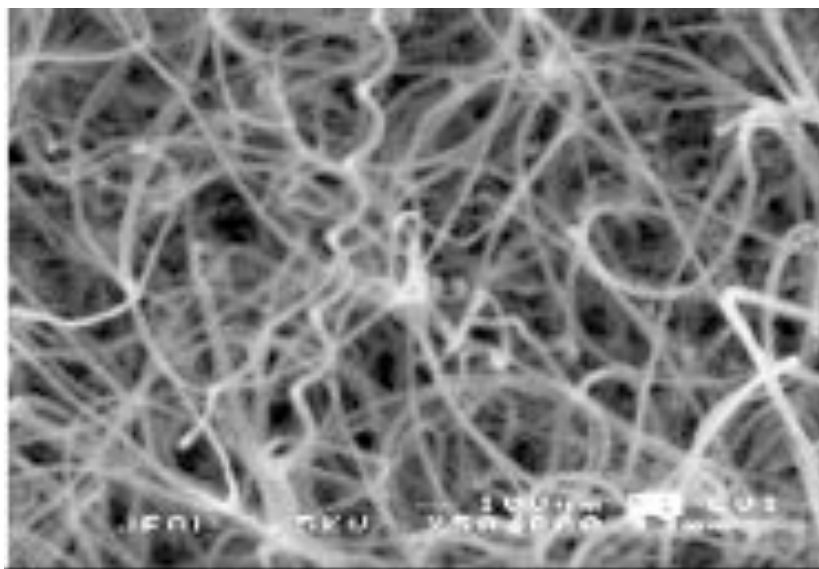


SWCNT



MWCNT

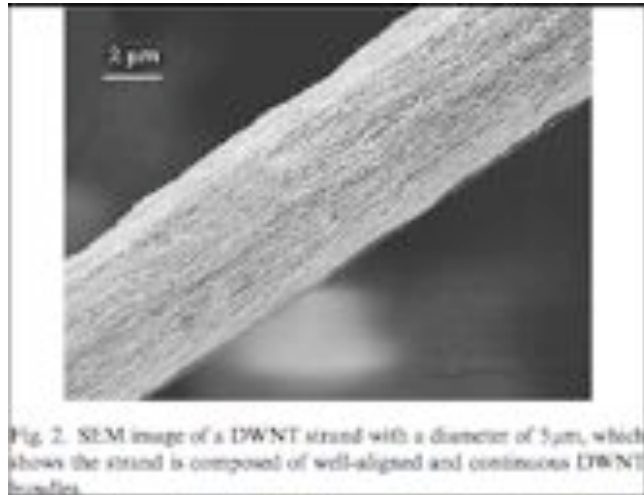
CNTs form as hollow single wall (SWCNT), double wall (DWCNT) and multiwalled MWCNT structures that range from 3 to 20 nm in diameter and about 300 nm in length.



Initially produced as irregular spaghetti-like CNTs with defects that cause the curvature and/or kinking.

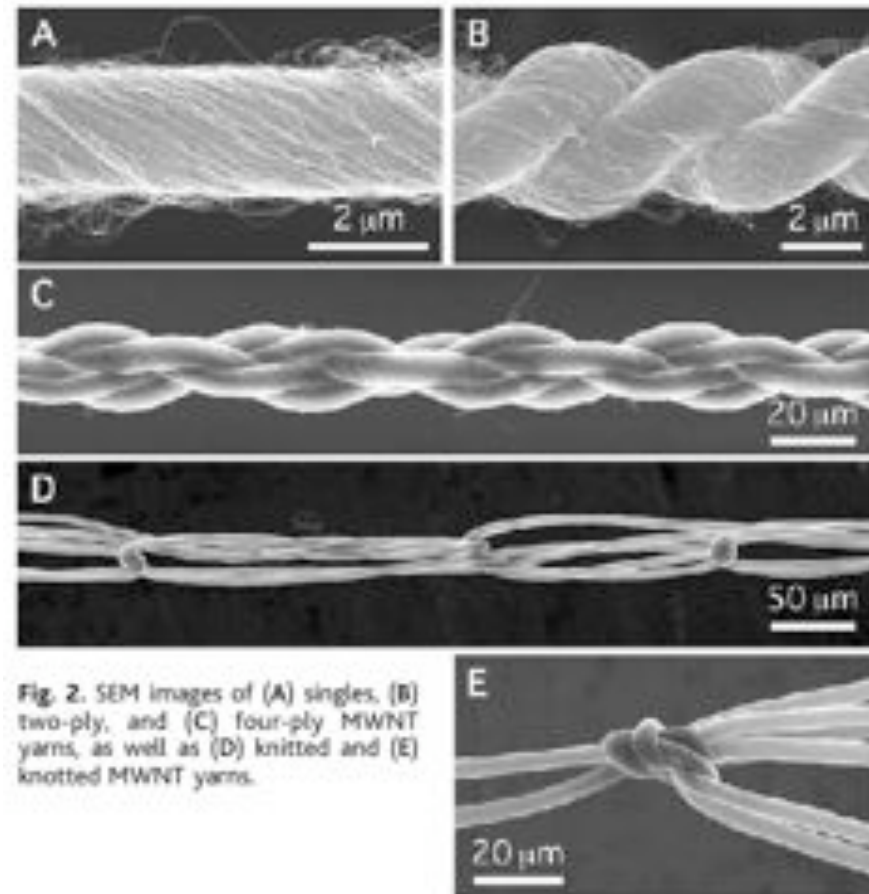
# Tremendous advances in creation of thick strands of CNTs

Tsinghua U. Beijing



By 2005 facilities in the US, China and England were producing nicely aligned MWCNT strands and rope-like cable containing about a trillion CNTs in cross section ranging from 2-20 μm in diameter

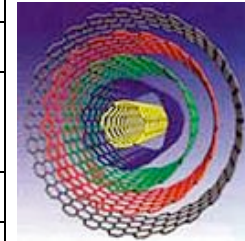
U. Texas



## ... and what are the properties of these CNTs?

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Ref	Object	strength GPa	stiffness GPa	Comments
This paper		50-200	1000	Theoretical
Yu et al.	MWCNT	11- <b>63</b> * 1.4-2.9**	270- <b>950</b> * 18-69**	*only outer wall area used ** entire cross section area
Pan et al.	MWNCT	1.72±0.64	490±230	
Li et al.	SWCNT	11.5		
Zhu et al.	SWNT	~1.0	49-77	strands
Y. Li et al.	DWCNT	1.2	16	strands



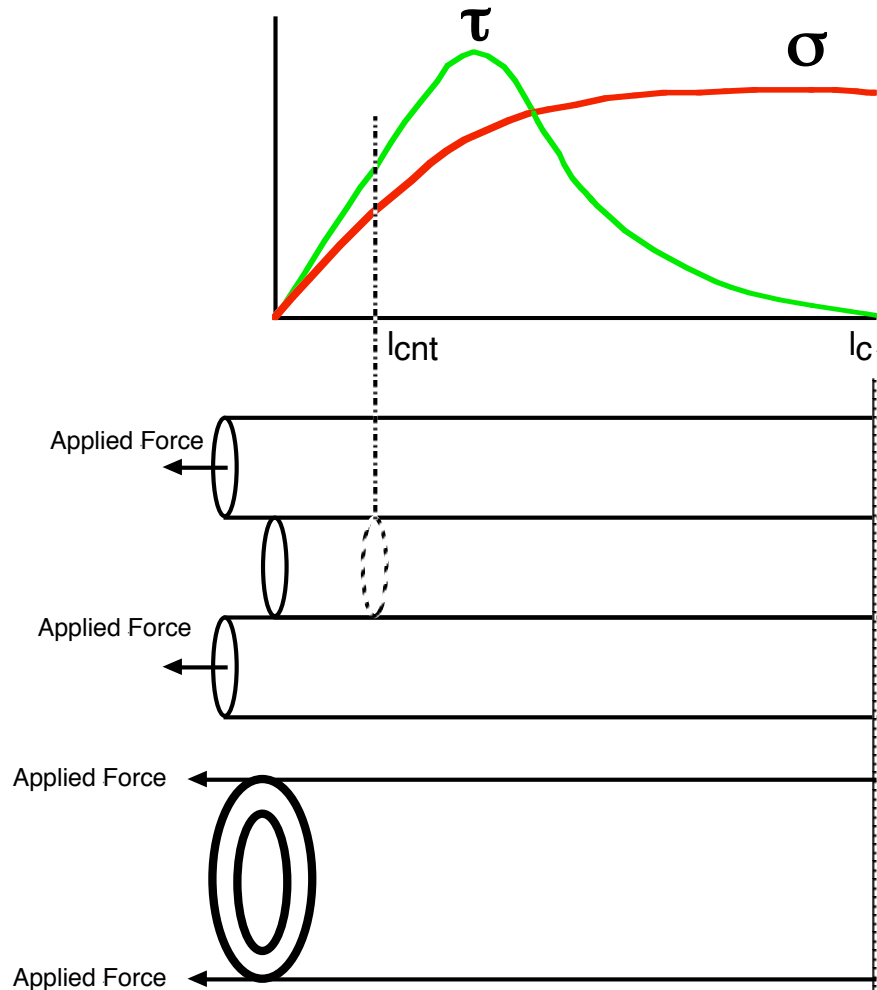
Yu et al. Science 2000;287;637-40  
 Pan et al. Appl Physics Lett 1999;74;3152-4  
 Li F. et al. Appl Phys Lett 2000;77;3161-3  
 Zhu et al. Science 2002;296;884-6  
 Y. Li et al Carbon 2005;43;31-35

**The strand strength of CNT is only 1.2 GPa (50 times less than theoretical!)**

**Even commercial carbon fibers have strengths ranging from 2-6 GPa.**

**Let's try to understand why the strength is so low ...**

## Frictional shear transfer of load between aligned CNTs is only partially successful



In a bundle of fibers the load transfers by the friction forces in shear parallel to the fiber surface.

**The measured shear modulus of CNTs is only 1 GPa vs 1000 GPa in tension.**

**The critical distance along the unbroken fiber needed to transfer the load is called  $l_c = (\sigma_f/\tau_f) \cdot d_f$**

A molecular mechanics analysis of the pullout of a single SWCNT from within a bundle of six other CNTs gave

$$l_c = 4 \mu\text{m (assuming } \sigma_f \text{ of 11GPa)}$$

$$\text{Average } l_{cnt} = 0.3 \mu\text{m} - \ll l_c$$

**Load from one CNT to another is only partially transferred and full strength of the bundle is not realized.**

## Load transfer between CNTs in bundles is much improved by twisting

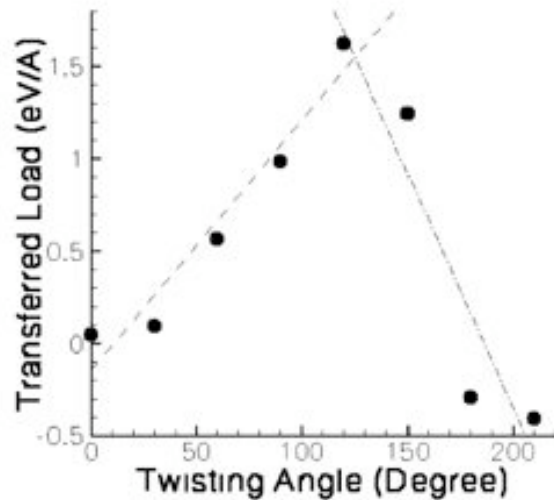


Fig. 9. Transferred load as a function of twisting angle (the dashed and dash-dotted lines are the linear fitting for the strengthening and weakening stage of the load transfer).

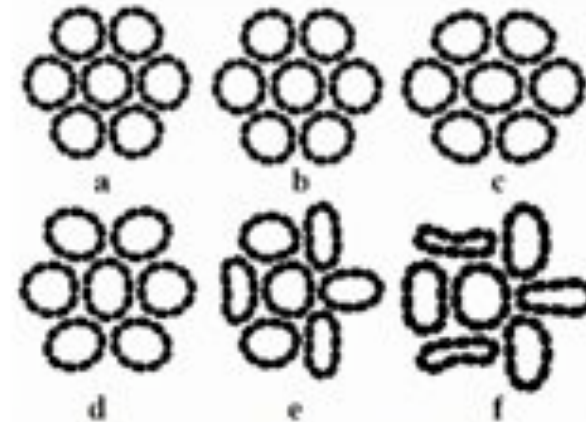


Fig. 8. Change in cross-section at the mid-length of the SWCNT bundle as a function of twist angle.

**Qian et al. Performed an Molecular Dynamics analysis on a twisted strand of 7 SWCNTs. The maximum axial load transferred was 1.7 GPa (1.55 eV/A) for the twist angle of 120 degrees (fibers  $\sim 10^\circ$  off axis). Twisting increases the pressure among fibers normal to the axis of loading and also increases in the frictional shear force. Beyond 120 degrees the distortion of the bundles resulted in lower load transfer.**

**So... a SWCNT bundle cannot be loaded to an axial stress much greater than a few GPa.**

# Vacancies\* have a deleterious effect on tensile strength

\*Vacancies introduced in manufacture or by cosmic rays in space

(Helsinki UoT studies)

**Vacancies in SWCNTs can form as a result of purification or irradiation. Single, double and triple vacancies were modeled with and MD code.**

**The reduction in stiffness and strength was modeled. Stiffness was reduced at most 3%.**

**Strength was reduced by 5-15 % depending on chirality of the SWCNT.**

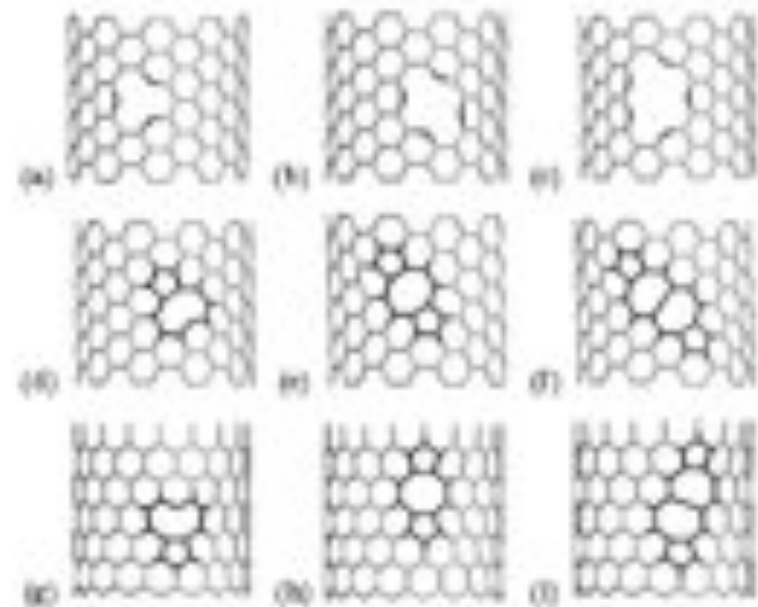


FIG. 2. Atomic networks of SWCNTs with non-reconstructed (a-c) and reconstructed (d-f) single (a,d,g), double (b,e,h) and triple (c,i,j) vacancies. Only the front wall of each tube is shown. The bottom row configurations correspond to a  $[11,0]$  zigzag SWNT, and the others to a  $[10,10]$  armchair SWNT.

## SW defects lower the strength (max strain) of CNTs

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In MD modeling Stone-Wales bond rotation occurs at 5% tensile strain.

Several such rotations then lead to local necking and tensile failure at strains of ~11%.

Even defect free CNTs can initiate such defects and fail at strains of ~10%

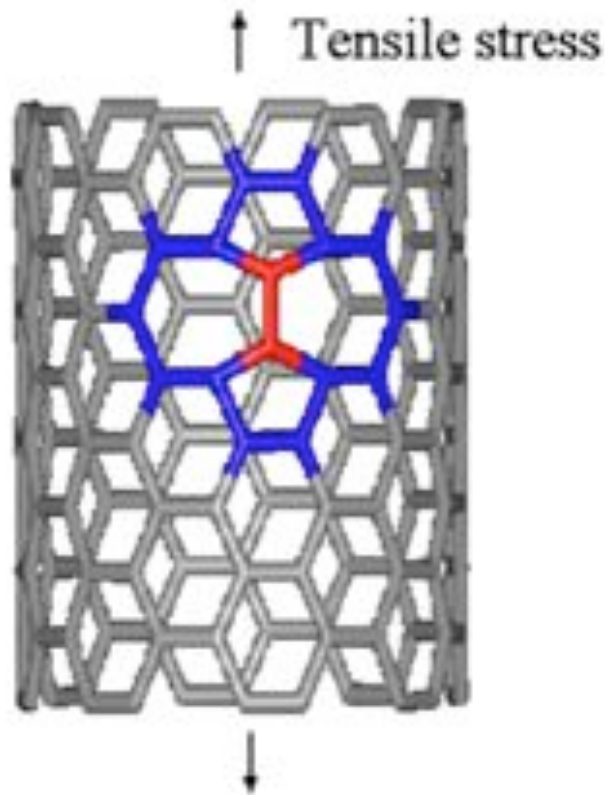


FIG. 1. A Stone-Wales bond rotation defect shown on an arm-chair CNT under tension.

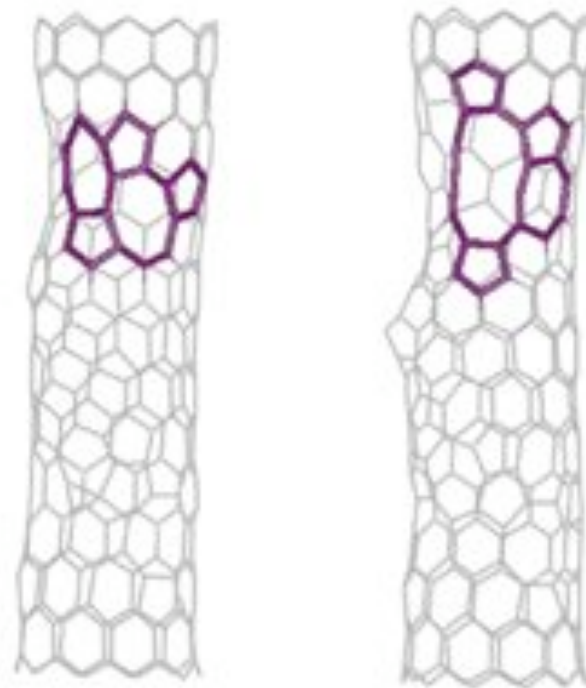


FIG. 3. The atomic configurations of the 11.5% strained CNT (10,0) near its yield strain at  $T = 1600$  K (shown is the section of the 60 Å CNT). Left: a group of pentagon and heptagon centered by an octagon which are resulted from several SW rotations in the connected region. Right: the followed of the start of the breaking of the CNT (10,0). Defects are marked in dark.

# **The proof of modeling is always in the experiment...**

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**In 2006 by stretching the nanotubes at high temperature, researchers at Boston College, Lawrence Livermore National Laboratory, and Massachusetts Institute of Technology have extended nanotubes by almost 280%.**

**"We discovered** the **superplasticity** phenomena accidentally," Jianyu Huang of Boston College told *nanotechweb.org*. "When my student Shuo Chen and I were studying the high-bias transport property of individual nanotubes, our scanning tunnelling microscopy (STM) probe was always attracted by the other electrode, which bent the nanotube and affected our measurement." As Chen and Huang pulled the STM probe away from the electrode, they discovered that the nanotube was much more stretchy than expected. To investigate the phenomenon further, the researchers made a 24 nm-long single-walled carbon nanotube by breaking down a multiwalled nanotube inside a high-resolution transmission electron microscope. They used a piezo manipulator to stretch the nanotube at a bias of 2.3 V. **The nanotube failed when it became 91 nm long, a tensile elongation of 280%. Its diameter at failure was 0.8 nm, 15 times smaller than its initial diameter of 12 nm.** The scientists reckon that the temperature inside the nanotube was more than 2000°C during deformation.

## **Conclusion:**

**Defects exist in carbon nanotubes and strength will be reduced from theoretical predictions – as is true in every other known material.**

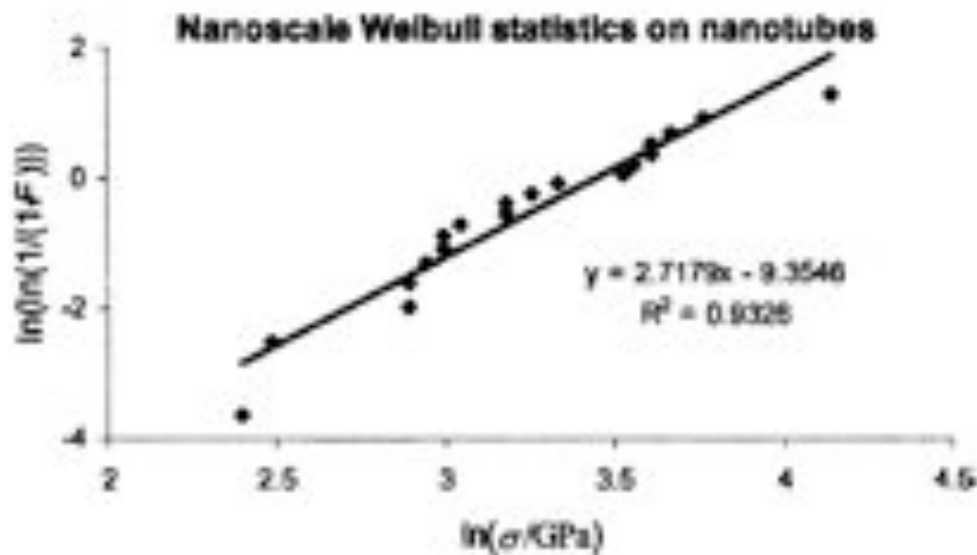
## Designing with Nanoscale Weibull Statistics

The strength of brittle materials is sensitive to the volumetric density of defects and has been found to follow a probability of failure model first proposed by Weibull (1951)

$P_f(s) = 1 - \exp[-n_v(\sigma/\sigma_{0v})^m]$  = probability of failure at stress level  $s$  and  $n_v$  is the number of defects in the volume of the test sample.

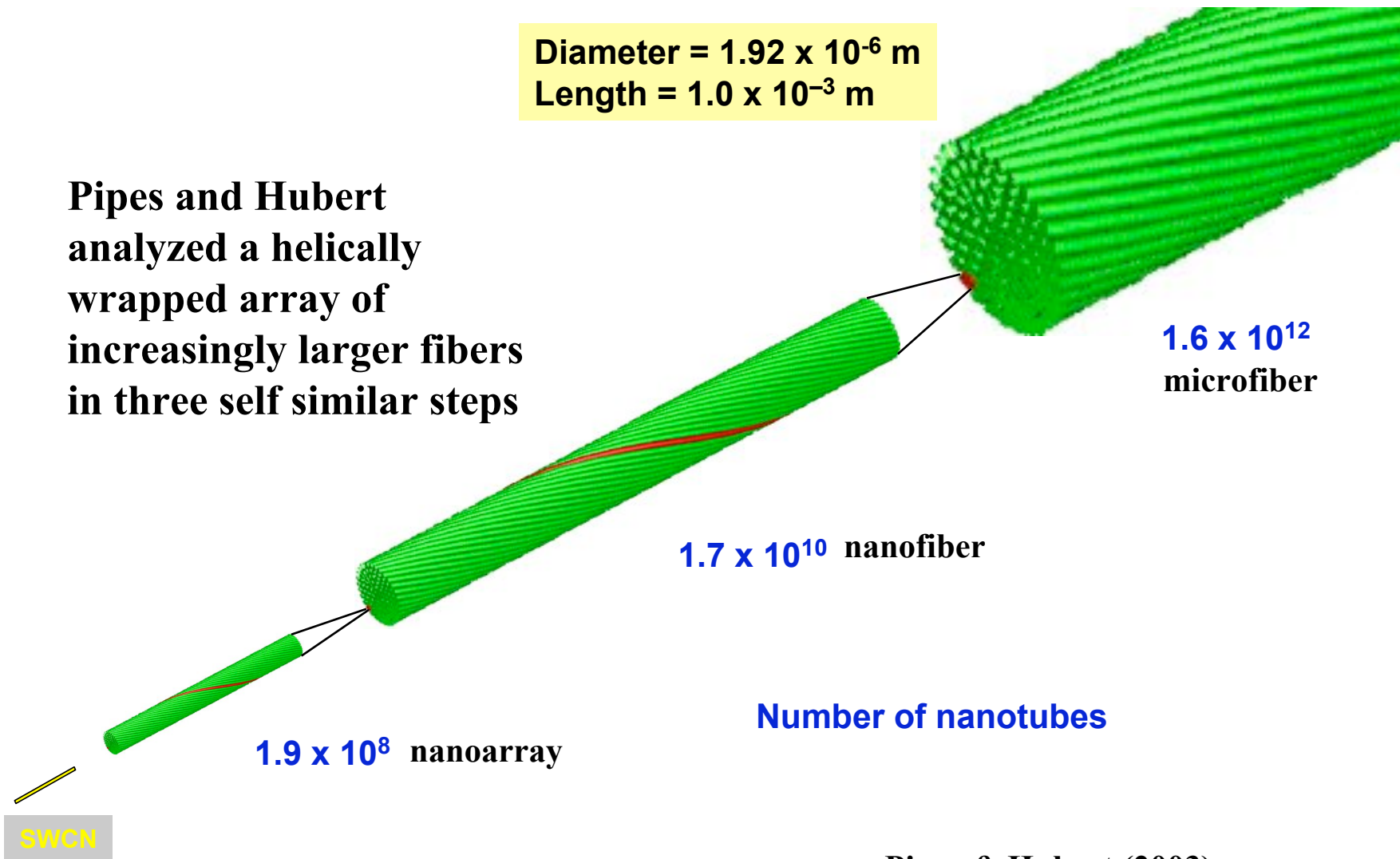
Pugno has extended this analysis to the outer wall failure of the MWCNTs tested by Yu et al (2000). He finds  $m = 3$  and  $\sigma_{0v} = 31$  Gpa.

**This means that there is a probability of 63% that the outer wall will fail at a stress level of 31 GPa**



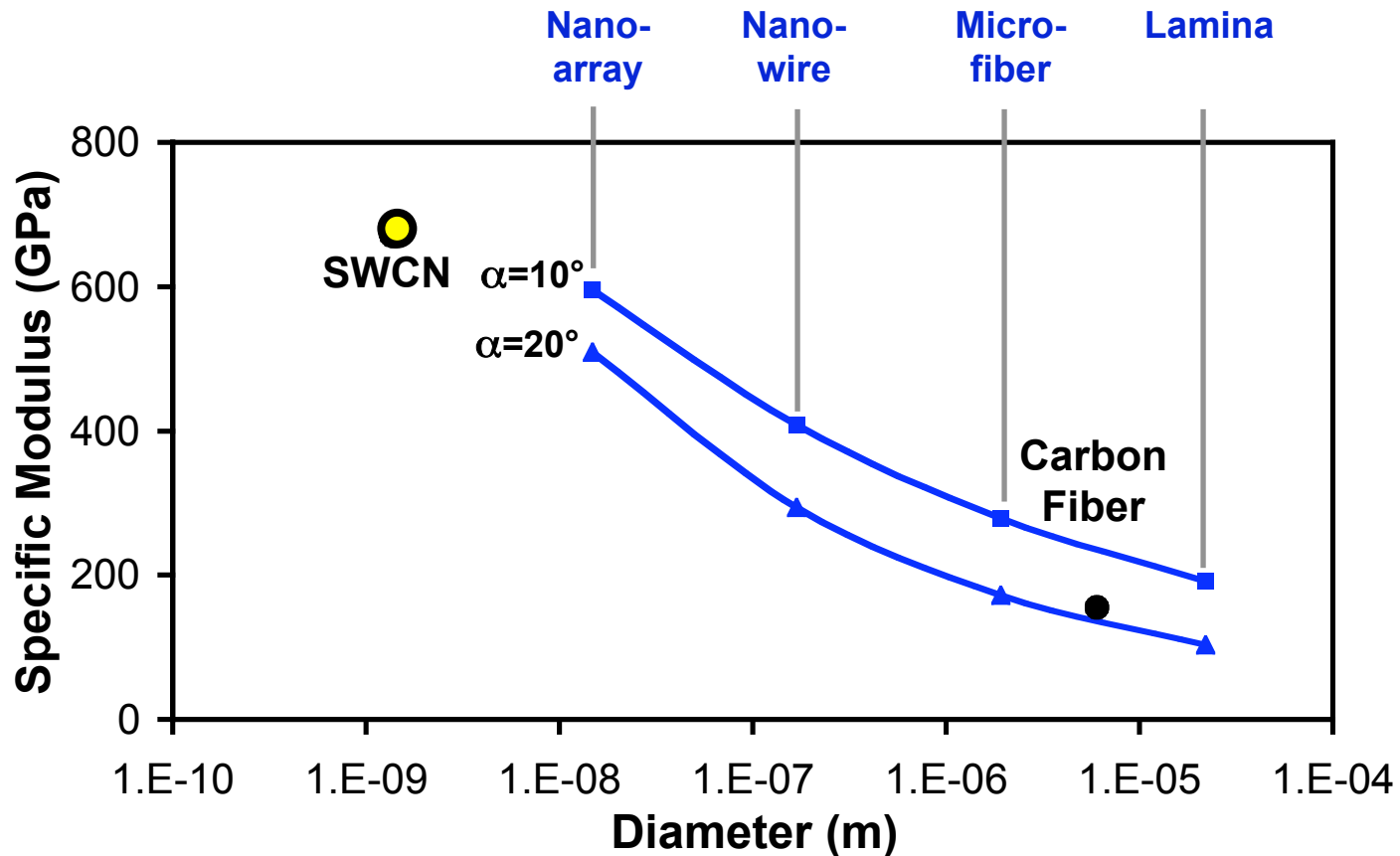
**Pugno argues: working stress for carbon nanotube “megacables” for the Space Elevator is at least 70% less than the theoretical strength of CNT**

# Creating a strand - 1 CNT to trillions



Pipes & Hubert (2003)

## But... CNT fibers loose stiffness at each buildup step



Pipes correctly reduces the stiffness of CNTs by accounting for the 40% by volume hollow in each SWCNT and for the space between each fiber in the successive self similar arrays and the angle of the fiber with respect to the axial load.

# Strength Reduction in Helical Hawser and Helical, Braided Ropes

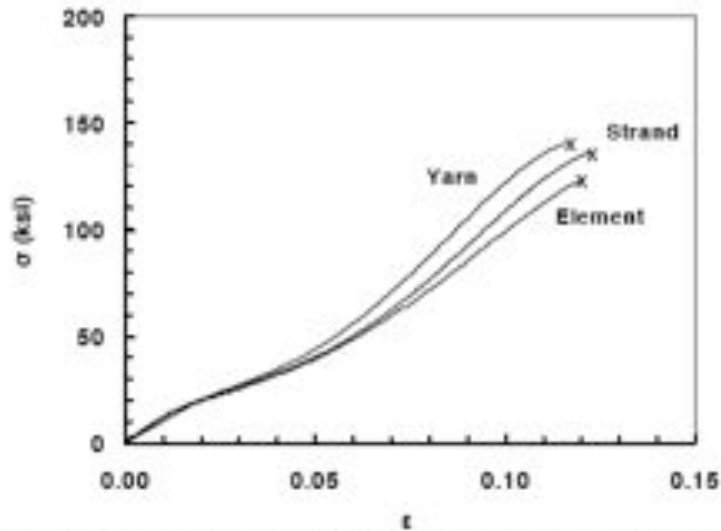


Figure 32 Comparison of stress-strain relationships among yarn, strand, and element



Figure 2 Definitions of strand, element and subrope (Mooring rope #1).

## University of Houston study 2002

A mechanics of failure of mooring (hawser) rope shows that there is a size effect of strength off about 5-10% at each element level. Stiffness at high stress is also reduced.

It is attributed to the difficulty of loading all fibers uniformly at increasing element size.



A survey of New England Rope's highest strength marine rope made from Spectra fiber shows:

**strength of 6-26 mm diameter helical, braided rope ranges from 26-32% of the published fiber strength of Spectra fiber.**

## CNT Conclusions

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- **CNTs contain defects that reduce strength well below theoretical**
- **The shear strength of CNTs is very low and makes it difficult to transfer load to interior CNTs in MWCNTs and to other CNTs arranged in arrays**
- **The highest strength values have been measured on walls of MWCNTs and do not account for the entire cross-section of the tube which is 40% by volume a void. When the proper cross section is used, the effective measured strength is only a few GPa**
- **Twisting the CNTs helps to transfer some load, but the very short length of CNTs compared to the critical shear length limits this to only a few GPa of load transfer**
- **As CNTs are built up to larger diameter structures there will be losses of stiffness and strength at each scale level.**

# Summary of the most likely materials for tether use

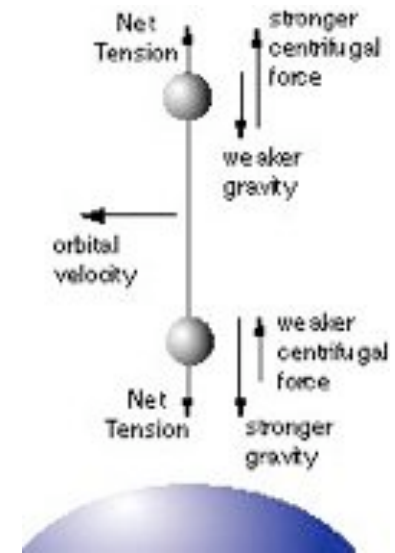
At this time the choice of fibers for a tether cable are among three that are very close in characteristic velocity:

- Zylon
- Spectra
- High Strength carbon fiber

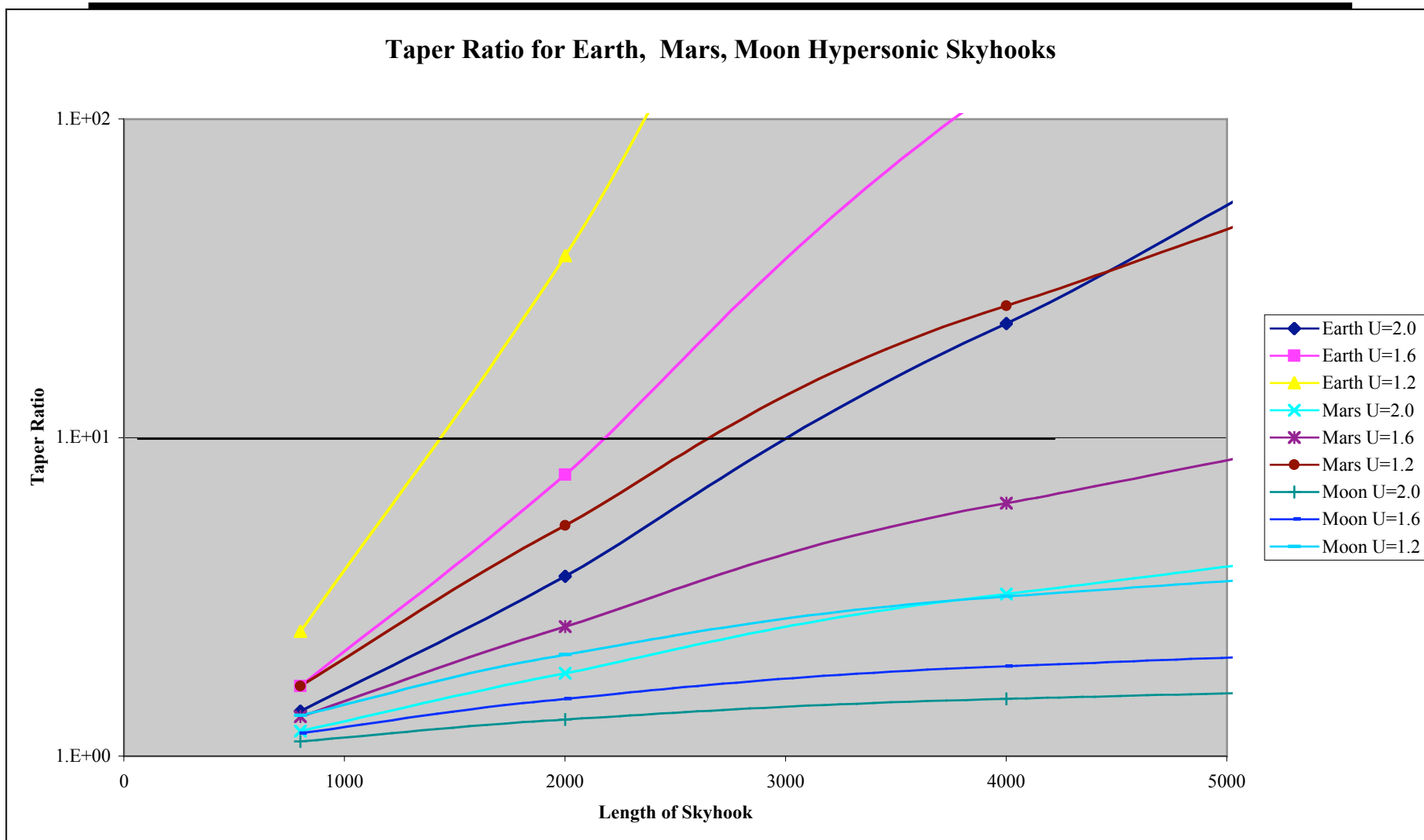
The best CNT strand to date is 1/2 the characteristic velocity of the best candidates

Table of characteristic velocities

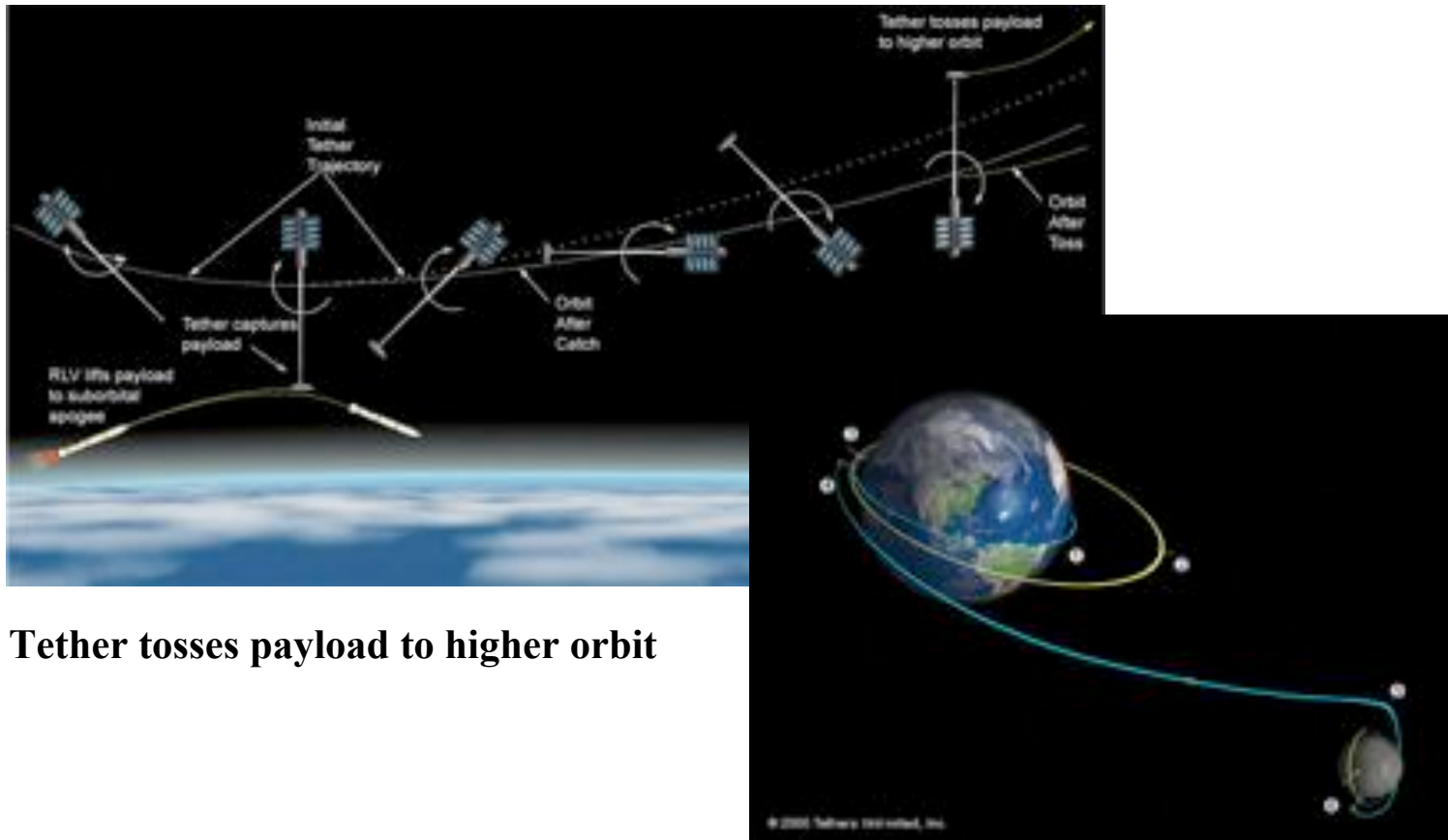
	Strength GPA	Sp. Grav.	Vc km/sec
CNT th	63	1.34	6.85674306
Zylon AS	5.8	1.54	1.94067869
Spectra 2000	3.5	0.97	1.89953874
C fiber	6	1.85	1.80090068
Kevlar	3.6	1.44	1.58113883
DWCNT	1.2	1.34	0.94632045



# A closer look at the taper ratio for shorter skyhooks



# A Multiple Rotovator Tether System

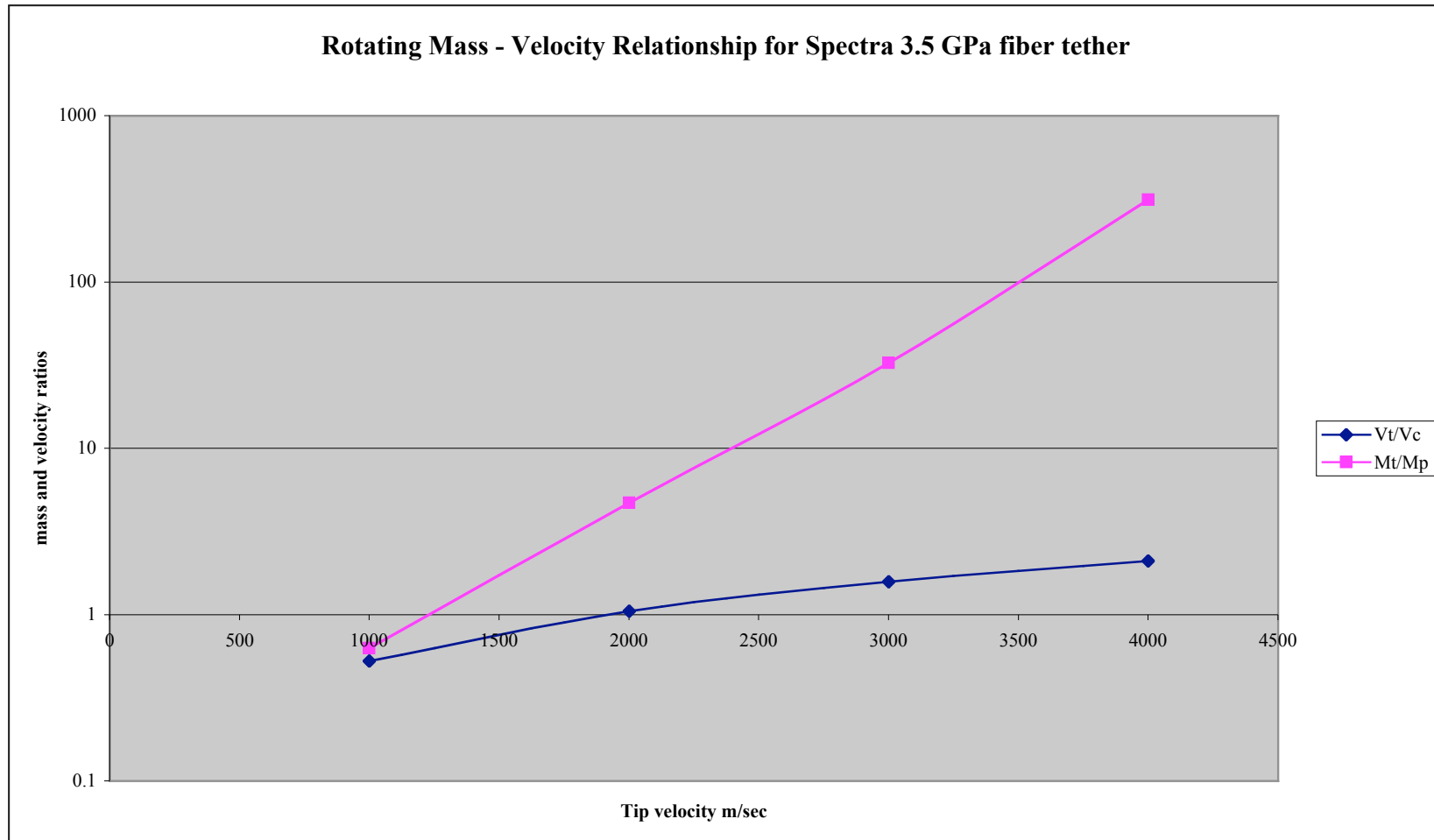


**Tether tosses payload to higher orbit**

From Tethers.com The **Cislunar Tether Transport System**. (1) A payload is launched into a LEO holding orbit; (2) A Tether Boost Facility in elliptical, equatorial Earth orbit picks up the payload (3) and tosses it (4) into a lunar transfer trajectory. When it nears the Moon, (5), a Lunavator Tether (6) captures it and delivers it to the lunar surface.

## Rotovator tether/payload mass ratio depends only on tip to characteristic velocity ratio

$m_t/m_p = \pi^{1/2} (V_t/V_c) \exp[(V_t/V_c)^2] \operatorname{erf}(V_t/V_c)$  in free space from the Moravec Eqn



# Near-Earth gravity effects require numerical solutions

## Tethers Unlimited Study (1999)

**Table 1 - Minimum Mass Ratio Rotovator™ Tether Parameters for HASTOL Application**

Tether Length	Orbital Radius	Orbital Velocity	Tip Velocity	Hypersonic Airplane Velocity		Tether to Payload Mass Ratio		
				$V_H = V_O - V_T - 470 \text{ m/s}$	Mach	Spectra™	2x	10x
L	$R_O$	$V_O$	$V_T$	(m/s)				
(km)	(km)	(m/s)	(m/s)	(m/s)	Mach			
400	6878	7614	2494	4650	15.0	10.4	2.4	0.37
500	6978	7559	2749	4340	14.0	16.7	4.2	0.56
600	7078	7506	3006	4030	13.0	27.1	5.9	0.65
700	7178	7453	3263	3720	12.0	44.0	8.2	0.73
800	7278	7402	3522	3410	11.0	71.8	11.6	0.90
900	7378	7352	3782	3100	10.0	117.6	16.3	1.07

**Rotovator tethers are met by spaceplanes with velocities of Mach 10-15 at an altitude of ~100 km . The tether will grab the payload and throw the payload into a higher orbit.**

## **Conclusions II - A break-through answer for Burt Rutan**

- **Momentum Exchange tethers above the Earth's atmosphere can be built with existing materials.**
- **The stationary and rotating tether concepts can be applied to Mars and Moon with even lower material working stress (characteristic velocity).**
- **Multiple tethers that hand off payloads to successively "higher" orbits offer the best prospect for a breakthrough technology to reduce the energy cost of reaching Earth orbit and beyond and allowing average citizen to afford the travel to orbital vacations or a new home off the planet Earth**

**And for you the audience:**

- **Don't let the vision of an Earth Space Elevator (that can't be built) blind you to other realistic tether solutions**

**Email: [crossmanf@earthlink.net](mailto:crossmanf@earthlink.net)**